

## Mathematical Analysis - List 9

- A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after (a) 1 s, (b) 3 s, and (c) 5 s. What can you conclude?
- Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is  $\pi/3$ .
- Show that  $f(x) = 2x + \cos x$  is a strictly monotonic function.
  - What is the value of  $f^{-1}(1)$ ?
  - Find  $(f^{-1})'(1)$ .
- Use l'Hospital's Rule to find the limit.
  - $\lim_{x \rightarrow \infty} \frac{\ln(2^x + 1)}{x}$ ;
  - $\lim_{x \rightarrow 1} \frac{\ln \sin \frac{\pi}{2}x}{\ln x}$ ;
  - $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x}}$ ;
  - $\lim_{x \rightarrow \infty} x \operatorname{arctg} x$ ;
  - $\lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^2}$ ;
  - $\lim_{x \rightarrow 1} \frac{x^{10} - 10x + 9}{x^5 - 5x + 4}$ ;
  - $\lim_{x \rightarrow 0^+} x \ln x$ ;
  - $\lim_{x \rightarrow 0^-} \left( \frac{1}{x} - \operatorname{ctg} x \right)$ ;
  - $\lim_{x \rightarrow 0} \frac{\ln \cos x}{\ln \cos 3x}$ ;
  - $\lim_{x \rightarrow \infty} \left( \frac{2}{\pi} \operatorname{arctg} x \right)^x$ ;
  - $\lim_{x \rightarrow 0^+} (1 + x)^{\ln x}$ ;
  - $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} \right)^{\sin x}$ .
- Find the intervals on which  $f$  is increasing or decreasing.
  - $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ ;
  - $f(x) = e^x(x + 1)$ ;
  - $f(x) = x - 3\sqrt[3]{x}$ ;
  - $f(x) = x \ln^2 x$ ;
  - $f(x) = x^3 - 30x^2 + 225x$ ;
  - $f(x) = xe^{-3x}$ .
- Find the local maximum and minimum values of  $f$ .
  - $f(x) = \frac{2x^2 - 1}{x^4}$ ;
  - $f(x) = x \ln x$ ;
  - $f(x) = x - \sqrt{x}$ ;
  - $f(x) = |x^2 - 5x - 6|$ ;
  - $f(x) = \frac{1}{x^2 - x}$ ;
  - $f(x) = x^3 - 4x^2$ ;
  - $f(x) = 2 \sin x + \cos 2x$ ;
  - $f(x) = (x - 5)e^x$ ;
  - $f(x) = \ln(1 + x^2)$ .
- Find the global maximum and global minimum values of  $f$  on the given interval.
  - $f(x) = 2x^3 - 15x^2 + 36x$ ,  $[1, 5]$ ;
  - $f(x) = \operatorname{arctg} \frac{1-x}{1+x}$ ,  $[0, 1]$ ;
  - $f(x) = (x - 3)^2 e^{|x|}$ ,  $[-1, 4]$ ;
  - $f(x) = 1 - |9 - x^2|$ ,  $[-5, 1]$ .
- Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius  $r$ .
- A right circular cylinder is inscribed in a sphere of radius  $r$ . Find the largest possible volume of such a cylinder.